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Analysis of Geometrically Non-Linear Thin Laminated Shells for Morphing Applications

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Abstract

In this work, a general model to describe the multistability of geometrically non-linear thin laminated shells is presented. The governing equations are cast for a set of curvilinear, non-orthogonal coordinates and are exact within the range of small strains, moderate rotations, and under the Kirchhoff kinematic assumption. The expressions of membrane strains and reference surface curvatures are identical to those used in the Sanders-Koiter-Leonard [Sanders JL., Quarterly of Applied Mathematics, 1962 - Koiter WT., Proc. Koninklijke Nederlandse Akademie van Wetenschappen, 1966 - Leonard RW., Ph.D. thesis, Virginia Polytechnic Institute, 1961] geometrical non-linear shell theories. A new set of compatibility conditions that take into account non-linear bending effects is presented. The compatibility conditions have been obtained by applying Koiter's small strain approximation [Koiter WT., Proc. of IUTAM Symposium on the theory of thin elastic shells, 1960] to the Riemann tensor, expressed in terms of first and second fundamental form of the shell reference surface [Gol'denveizer AL., Pergamon Press, 1961 - Mushtari KM., Galimov KZ., NASA-TT-F62, 1961]. As a result, the two Codazzi-Mainardi equations and Gauss's compatibility condition have been obtained. In particular, Gauss's compatibility condition is non-linear in terms of curvatures, as opposed to previous works [Nemeth MP., NASA/TM-217964, 2013], that appear to be inconsistent with a geometrically non-linear shell theory.

In a recent work [Lamacchia E., Pirrera A., Chenchiah I.V. and Weaver P.M., Composite Structures, 2015], the authors showed that for a sufficiently shallow shell, the membrane and bending effects can be decoupled using the semi-inverse formulation of the constitutive equations, and the shell can be described as an initially curved plate by assuming von Kármán kinematics. However, for general doubly-curved shells, the membrane and bending effects are coupled due to the surface curvature. Therefore, both Gauss's and Codazzi-Mainardi equations are function of membrane and bending strains, as opposed to von Kármán plates, in which the Gaussian curvature is the divergence of the membrane strain, and the Codazzi-Mainardi equations link the derivative of the bending strains along the coordinates lines [Lamacchia E., Pirrera A., Chenchiah I.V. and Weaver P.M., Composite Structures, 2015].

In this work, any simplifying assumption on the shell initial curvature is dropped. The multistability of thin laminated shell is described in a novel, compatibility-based approach. The key step of the proposed model is to exploit the membrane/bending coupling in doubly-curved shells by combining Gauss's compatibility condition and the two Codazzi-Mainardi equations in order to find a strain field function of the surface change of curvature. As a result, the total strain energy can be expressed as a function of the curvatures only. The

equilibria configurations are obtained by minimising the total strain energy with respect to the curvatures field.

The governing equations are recast with a suitable set of differential operators and discretised using the Differential Quadrature Method (DQM). By doing so, the computational cost is kept at relatively low levels, as the strain field can be obtained once and for all by inverting a small (compared to FE) matrix of DQM weighting coefficients.

The proposed general shell model as been has been benchmarked against exemplar case studies taken from the literature. It shows an increment of accuracy of one order of magnitude when compared to the solution given by assuming von Kármán kinematics [Lamacchia E., Pirrera A., Chenchiah I.V. and Weaver P.M., Composite Structures, 2015]. This increment of accuracy is due to the presence of additional terms in the shell kinematic equations that are a function of the surface radii of curvature and torsion. These additional terms, despite being small, contribute considerably to the refinement of the solution.